Materials Processing of Semiconductor (반도체재료공정학)

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Basic Plasma Equations and Equilibrium

Basic Plasma Equations and Equilibrium

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M.A. Lieberman and A.J. Lichtenberg, Principles of Plasma Discharges and Materials Processing, 2nd ed. (John Wiley, New York, 2005).

Basic Plasma Equations and Equilibrium

Introduction

The plasma medium is complicated in that the charged particles are both affected by external electric and magnetic fields and contribute to them. The resulting self-consistent system is nonlinear and very difficult to analyze. Furthermore, the interparticle collisions, although also electromagnetic in character, occur on space and time scales that are usually much shorter than those of the applied fields or the fields due to the average motion of the particles.

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Basic Plasma Equations and Equilibrium

Introduction

To make progress with such a complicated system, various simplifying approximations are needed. The interparticle collisions are considered independently of the larger scale fields to determine an equilibrium distribution of the charged-particle velocities. The velocity distribution is averaged over velocities to obtain the macroscopic motion. The macroscopic motion takes place in external applied fields and in the macroscopic fields generated by the average particle motion.

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Basic Plasma Equations and Equilibrium

Introduction

These self-consistent fields are nonlinear, but may be linearized in some situations, particularly when dealing with waves in plasmas. The effect of spatial variation of the distribution function leads to pressure forces in the macroscopic equations. The collisions manifest themselves in particle generation and loss processes, as an average friction force between different particle species, and in energy exchanges among species.

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Basic Plasma Equations and Equilibrium

Introduction

In this chapter, we consider the basic equations that govern the plasma medium, concentrating attention on the macroscopic system. The complete derivation of these equations, from fundamental principles, is beyond the scope of the text. We shall make the equations plausible and, in the easier instances, supply some derivations in appendices. For the reader interested in more rigorous treatment, references to the literature will be given.

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Basic Plasma Equations and Equilibrium

Introduction

In Section 2.2, we introduce the macroscopic field equations and the current and voltage. In Section 2.3, we introduce the fundamental equation of plasma physics, for the evolution of the particle distribution function, in a form most applicable for weakly ionized plasmas. We then define the macroscopic quantities and indicate how the macroscopic equations are obtained by taking moments of the fundamental equation. References given in the text can be consulted for more details of the averaging procedure.

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Although the macroscopic equations depend on the equilibrium distribution, their form is independent of the equilibrium. To solve the equations for particular problems the equilibrium must be known. In Section 2.4, we introduce the equilibrium distribution and obtain some consequences arising from it and from the field equations. The form of the equilibrium distribution will be shown to be a consequence of the interparticle collisions, in Appendix B.

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Basic Plasma Equations and Equilibrium

FIELD EQUATIONS, CURRENT, AND VOLTAGE

Maxwell's Equations

The usual macroscopic form of Maxwell's equations are

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \tag{2.2.1}$$

$$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J} \tag{2.2.2}$$

$$\epsilon_0 \nabla \cdot \mathbf{E} = \rho \tag{2.2.3}$$

and

$$\mu_0 \nabla \cdot \mathbf{H} = 0 \tag{2.2.4}$$

where **E(r, *t*)** and **H(r, *t*)** are the electric and magnetic field vectors and where μ_0=4 $\pi\times$ 10^7 H/m and ϵ_0 = 8.854 $\times$ 10^{-12} F/m are the permeability and permittivity of free space.

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Basic Plasma Equations and Equilibrium

FIELD EQUATIONS, CURRENT, AND VOLTAGE

Maxwell's Equations

The sources of the fields, the charge density **r(r,** t**)** and the current density **J(r,** t**)**, are related by the charge continuity equation (Problem 2.1):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0 \qquad (2.2.5)$$

In general,

$$\mathbf{J} = \mathbf{J}_{\text{cond}} + \mathbf{J}_{\text{pol}} + \mathbf{J}_{\text{mag}}$$

where the conduction current density $\mathbf{J}_{cond}$ is due to the motion of the free charges, the polarization current density $\mathbf{J}_{pol}$ is due to the motion of bound charges in a dielectric material, and the magnetization current density $\mathbf{J}_{mag}$ is due to the magnetic moments in a magnetic material. In a plasma in vacuum, $\mathbf{J}_{pol}$ and $\mathbf{J}_{mag}$ are zero and $\mathbf{J} = \mathbf{J}_{cond}$.

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FIELD EQUATIONS, CURRENT, AND VOLTAGE

Maxwell's Equations

If (2.2.3) is integrated over a volume $\mathcal{V}$, enclosed by a surface $\mathcal{S}$, then we obtain its integral form, Gauss' law:

$$\epsilon_0 \oint_{\mathcal{S}} \mathbf{E} \cdot d\mathbf{A} = q \tag{2.2.6}$$

where q is the total charge inside the volume. Similarly, integrating (2.2.5), we obtain

$$\frac{dq}{dt} + \oint_{\mathcal{S}} \mathbf{J} \cdot d\mathbf{A} = 0$$

which states that the rate of increase of charge inside $\mathcal{V}$ is supplied by the total current flowing across $\mathcal{S}$ into $\mathcal{V}$, that is, that charge is conserved.

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Basic Plasma Equations and Equilibrium

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Maxwell's Equations

In (2.2.2), the first term on the RHS is the displacement current density flowing in the vacuum, and the second term is the conduction current density due to the free charges. We can introduce the total current density

$$\mathbf{J}_T = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J} \tag{2.2.7}$$

and taking the divergence of (2.2.2), we see that

$$\nabla \cdot \mathbf{J}_T = 0 \tag{2.2.8}$$

In one dimension, this reduces to $dJ_{Tx}/dx = 0$, such that $J_{Tx} = J_{Tx}(t)$, independent of x. Hence, for example, the total current flowing across a spatially nonuniform

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Maxwell's Equations

one-dimensional discharge is independent of x, as illustrated in Figure 2.1. A generalization of this result is *Kirchhoff's current law*, which states that the sum of the currents entering a *node*, where many current-carrying conductors meet, is zero. This is also shown in Figure 2.1, where $I_{rf} = I_T + I_1$.

If the time variation of the magnetic field is negligible, as is often the case in plasmas, then from Maxwell's equations $\nabla \times \mathbf{E} \approx 0$. Since the curl of a gradient is zero, this implies that the electric field can be derived from the gradient of a scalar potential,

$$\mathbf{E} = -\nabla\Phi \qquad (2.2.9)$$

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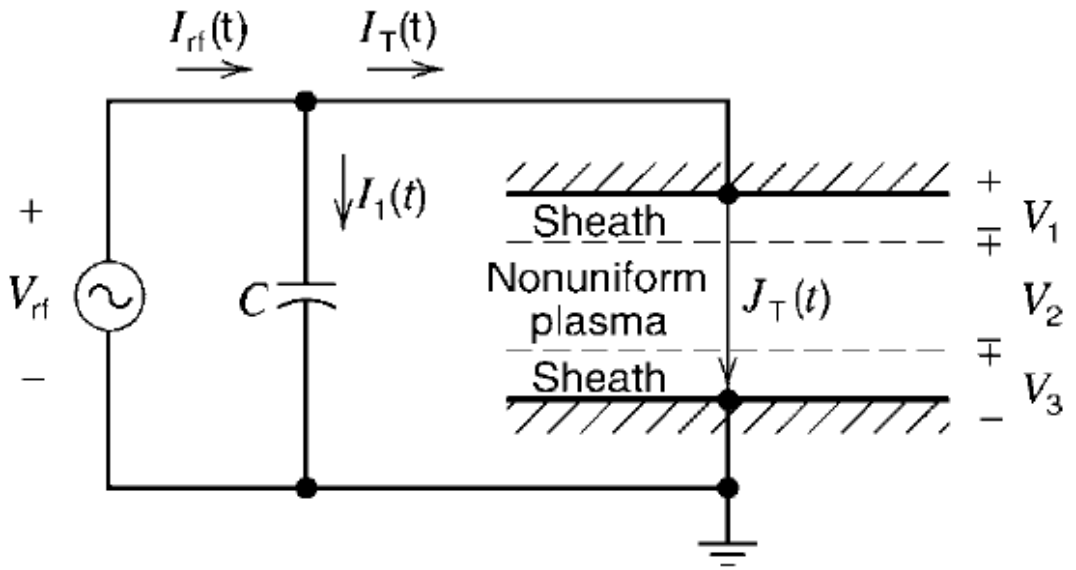


FIGURE 2.1. Kirchhoff's circuit laws: The total current J_T flowing across a nonuniform one-dimensional discharge is independent of x; the sum of the currents entering a node is zero ($I_{rf} = I_T + I_1$); the sum of voltages around a loop is zero ($V_{rf} = V_1 + V_2 + V_3$).

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Basic Plasma Equations and Equilibrium

FIELD EQUATIONS, CURRENT, AND VOLTAGE

Integrating (2.2.9) around any closed loop $\mathcal{C}$ gives

$$\oint_{\mathcal{C}} \mathbf{E} \cdot d\ell = -\oint_{\mathcal{C}} \nabla\Phi \cdot d\ell = -\oint_{\mathcal{C}} d\Phi = 0 \qquad (2.2.10)$$

Hence, we obtain *Kirchhoff's voltage law*, which states that the sum of the voltages around any loop is zero. This is illustrated in Figure 2.1, for which we obtain

$$V_{\mathrm{rf}} = V_1 + V_2 + V_3$$

that is, the source voltage V_{rf} is equal to the sum of the voltages V_1 and V_3 across the two sheaths and the voltage V_2 across the bulk plasma. Note that currents and voltages can have positive or negative values; the directions for which their values are designated as positive must be specified, as shown in the figure.

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Basic Plasma Equations and Equilibrium

FIELD EQUATIONS, CURRENT, AND VOLTAGE

If (2.2.9) is substituted in (2.2.3), we obtain

$$\nabla^2\Phi = -\frac{\rho}{\epsilon_0} \tag{2.2.11}$$

Equation (2.2.11), *Poisson's equation*, is one of the fundamental equations that we shall use. As an example of its application, consider the potential in the center ($x = 0$) of two grounded ($\Phi = 0$) plates separated by a distance $l = 10$ cm and containing a uniform ion density $n_i = 10^{10}$ cm^{-3}, without the presence of neutralizing electrons. Integrating Poisson's equation

$$\frac{d^2\Phi}{dx^2} = -\frac{en_i}{\epsilon_0}$$

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using the boundary conditions that $\Phi = 0$ at $x = \pm l/2$ and that $d\Phi/dx = 0$ at $x = 0$ (by symmetry), we obtain

$$\Phi = \frac{1}{2}\frac{en_i}{\epsilon_0}\left[\left(\frac{l}{2}\right)^2 - x^2\right]$$

The maximum potential in the center is 2.3×10^5 V, which is impossibly large for a real discharge. Hence, the ions must be mostly neutralized by electrons, leading to a quasi-neutral plasma.

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FIELD EQUATIONS, CURRENT, AND VOLTAGE

Lorentz Force Equation

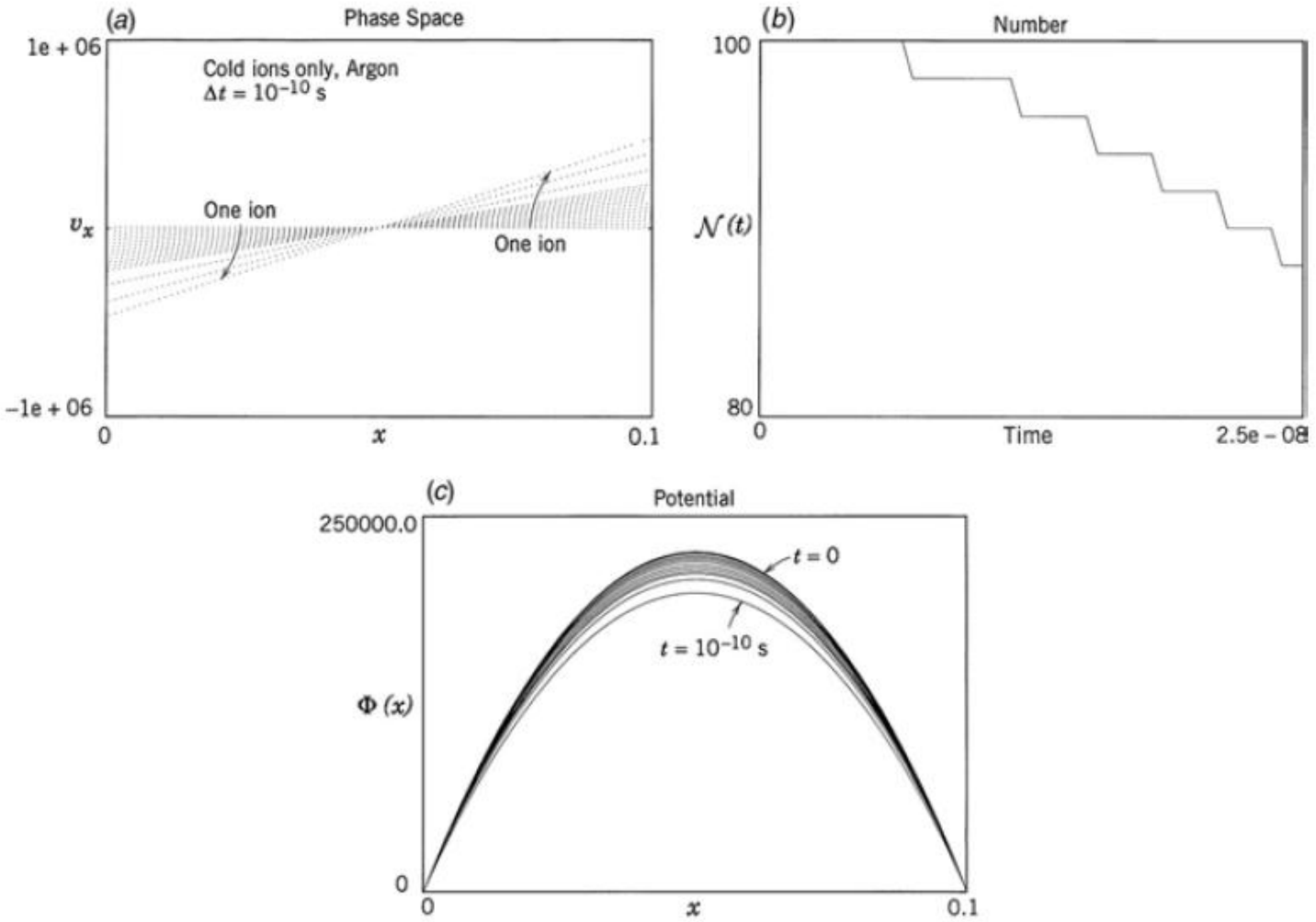
Figure 2.2 shows a PIC simulation time history over 10^{-10} s of (*a*) the v_x–x phase space, (*b*) the number $\mathcal{N}$ of sheets versus time, and (*c*) the potential Φ versus x for 100 unneutralized ion sheets (with e/M for argon ions). We see the ion acceleration in (*a*), the loss of ions in (*b*), and the parabolic potential profile in (*c*); the maximum potential decreases as ions are lost from the system. We consider quasi-neutrality further in Section 2.4.

Electric and magnetic fields exert forces on charged particles given by the *Lorentz force law*:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad (2.2.12)$$

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where **v** is the particle velocity and $\mathbf{B} = \mu_0\mathbf{H}$ is the *magnetic induction vector*. The charged particles move under the action of the Lorentz force. The moving charges in turn contribute to both ρ and **J** in the plasma. If ρ and **J** are linearly related to **E** and **B**, then the field equations are linear. As we shall see, this is not generally the case for a plasma. Nevertheless, linearization may be possible in some cases for which the plasma may be considered to have an *effective dielectric constant*; that is, the "free charges" play the same role as "bound charges" in a dielectric. We consider this further in Chapter 4.

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Basic Plasma Equations and Equilibrium

THE CONSERVATION EQUATIONS

Boltzmann's Equation

For a given species, we introduce a *distribution function* $f(\mathbf{r}, \mathbf{v}, t)$ in the six-dimensional *phase space* $(\mathbf{r}, \mathbf{v})$ of particle positions and velocities, with the interpretation that

$$f(\mathbf{r}, \mathbf{v}, t)\mathrm{d}^3 r\, \mathrm{d}^3 v = \text{number of particles inside a six-dimensional phase space volume } \mathrm{d}^3 r\, \mathrm{d}^3 v \text{ at } (\mathbf{r}, \mathbf{v}) \text{ at time } t$$

The six coordinates $(\mathbf{r}, \mathbf{v})$ are considered to be independent variables. We illustrate the definition of f and its phase space in one dimension in Figure 2.3. As particles drift in phase space or move under the action of macroscopic forces, they flow into or out of the fixed volume $\mathrm{d}x\, \mathrm{d}v_x$. Hence the distribution function f should obey a continuity equation which can be derived as follows. In a

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Basic Plasma Equations and Equilibrium

THE CONSERVATION EQUATIONS

Boltzmann's Equation

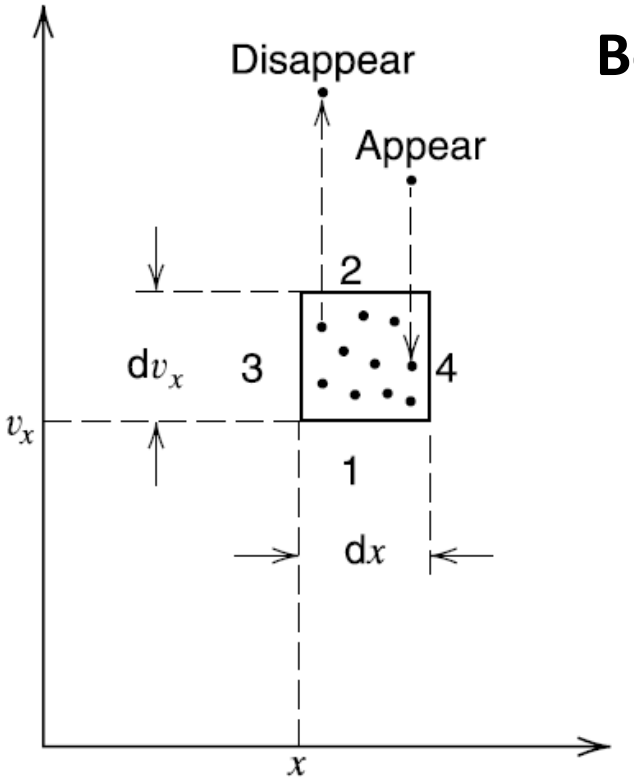


FIGURE 2.3. One-dimensional v_x–x phase space, illustrating the derivation of the Boltzmann equation and the change in f due to collisions.

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Basic Plasma Equations and Equilibrium

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time dt,

$f(x, v_x, t)\,dx\,a_x(x, v_x, t)\,dt$ particles flow into $dx\,dv_x$ across face 1

$f(x, v_x + dv_x, t)\,dx\,a_x(x, v_x + dv_x, t)\,dt$ particles flow out of $dx\,dv_x$ across face 2

$f(x, v_x, t)\,dv_x\,v_x\,dt$ particles flow into $dx\,dv_x$ across face 3

$f(x + dx, v_x, t)\,dv_x\,v_x\,dt$ particles flow out of $dx\,dv_x$ across face 4

where $a_x v dv_x/dt$ and $v_x \equiv dx/dt$ are the flow velocities in the v_x and x directions, respectively. Hence

$$
\begin{aligned}
&f(x, v_x, t + dt)\,dx\,dv_x - f(x, v_x, t)\,dx\,dv_x \\
&\quad = [f(x, v_x, t)a_x(x, v_x, t) - f(x, v_x + dv_x, t)a_x(x, v_x + dv_x, t)]\,dx\,dt \\
&\qquad + [f(x, v_x, t)v_x - f(x + dx, v_x, t)v_x]\,dv_x\,dt
\end{aligned}
$$

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THE CONSERVATION EQUATIONS

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Dividing by $dx\,dv_x\,dt$, we obtain

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial x}(f v_x) - \frac{\partial}{\partial v_x}(f a_x) \tag{2.3.1}$$

Noting that v_x is independent of x and assuming that the acceleration $a_x = F_x/m$ of the particles does not depend on v_x, then (2.3.1) can be rewritten as

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + a_x \frac{\partial f}{\partial v_x} = 0$$

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Collisionless Boltzmann Equation / Vlasov Equation

The three-dimensional generalization,

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} f + \mathbf{a} \cdot \nabla_{\mathbf{v}} f = 0 \qquad (2.3.2)$$

with $\nabla_{\mathbf{r}} = (\hat{x}\,\partial/\partial x + \hat{y}\,\partial/\partial y + \hat{z}\,\partial/\partial z)$ and $\nabla_{\mathbf{v}} = (\hat{x}\,\partial/\partial v_x + \hat{y}\,\partial/\partial v_y + \hat{z}\,\partial/\partial v_z)$ is called the *collisionless Boltzmann equation* or *Vlasov equation*.

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THE CONSERVATION EQUATIONS

Boltzmann's Equation

In addition to flows into or out of the volume across the faces, particles can "suddenly" appear in or disappear from the volume due to very short time scale interparticle collisions, which are assumed to occur on a timescale shorter than the evolution time of f in (2.3.2). Such collisions can practically instantaneously change the velocity (but not the position) of a particle. Examples of particles suddenly appearing or disappearing are shown in Figure 2.3. We account for this effect, which changes f, by adding a "collision term" to the right-hand side of (2.3.2), thus obtaining the *Boltzmann equation*:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} f + \frac{\mathbf{F}}{m} \cdot \nabla_{\mathbf{v}} f = \left.\frac{\partial f}{\partial t}\right|_{\mathrm{c}} \qquad (2.3.3)$$

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FIELD EQUATIONS, CURRENT, AND VOLTAGE

Boltzmann's Equation

The collision term in integral form will be derived in Appendix B. The preceding heuristic derivation of the Boltzmann equation can be made rigorous from various points of view, and the interested reader is referred to texts on plasma theory, such as Holt and Haskel (1965). A kinetic theory of discharges, accounting for non-Maxwellian particle distributions, must be based on solutions of the Boltzmann equation. We give an introduction to this analysis in Chapter 18.

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Basic Plasma Equations and Equilibrium

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Macroscopic Quantities

The complexity of the dynamical equations is greatly reduced by averaging over the velocity coordinates of the distribution function to obtain equations depending on the spatial coordinates and the time only. The averaged quantities, such as species density, mean velocity, and energy density are called macroscopic quantities, and the equations describing them are the macroscopic conservation equations. To obtain these averaged quantities we take velocity moments of the distribution function, and the equations are obtained from the moments of the Boltzmann equation.

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Basic Plasma Equations and Equilibrium

THE CONSERVATION EQUATIONS

The average quantities that we are concerned with are the particle density,

Macroscopic Quantities

$$n(\mathbf{r},t) = \int f \, \mathrm{d}^3 v \tag{2.3.4}$$

the particle flux

$$\Gamma(\mathbf{r},t) = n\mathbf{u} = \int \mathbf{v} f \, \mathrm{d}^3 v \tag{2.3.5}$$

where $\mathbf{u}(\mathbf{r},t)$ is the mean velocity, and the particle kinetic energy per unit volume

$$w = \frac{3}{2}p + \frac{1}{2}mu^2 n = \frac{1}{2}m \int v^2 f \, \mathrm{d}^3 v \tag{2.3.6}$$

where $p(\mathbf{r},t)$ is the isotropic pressure, which we define below. In this form, w is sum of the *internal* energy density $\frac{3}{2}p$ and the *flow* energy density $\frac{1}{2}mu^2 n$.

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Particle Conservation

The lowest moment of the Boltzmann equation is obtained by integrating all terms of (2.3.3) over velocity space. The integration yields the macroscopic *continuity equation*:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = G - L \qquad (2.3.7)$$

The collision term in (2.3.3), which yields the right-hand side of (2.3.7), is equal to zero when integrated over velocities, except for collisions that create or destroy

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THE CONSERVATION EQUATIONS

Particle Conservation

particles, designated as G and L, respectively (e.g., ionization, recombination). In fact, (2.3.7) is transparent since it physically describes the conservation of particles. If (2.3.7) is integrated over a volume $\mathcal{V}$ bounded by a closed surface $\mathcal{S}$, then (2.3.7) states that the net number of particles generated per second within $\mathcal{V}$, either flows across the surface $\mathcal{S}$ or increases the number of particles within $\mathcal{V}$. For common low-pressure discharges in the steady state, G is usually due to ionization by electron–neutral collisions:

$$G = \nu_{iz} n_e$$

where ν_{iz} is the ionization frequency. The volume loss rate L, usually due to recombination, is often negligible. Hence

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$$\nabla \cdot (n\mathbf{u}) = \nu_{iz} n_e \qquad (2.3.8)$$

in a typical discharge. However, note that the continuity equation is clearly not sufficient to give the evolution of the density n, since it involves another quantity, the mean particle velocity $\mathbf{u}$.

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Momentum Conservation

To obtain an equation for **u**, a first moment is formed by multiplying the Boltzmann equation by **v** and integrating over velocity. The details are complicated and involve evaluation of tensor elements. The calculation can be found in most plasma theory texts, for example, Krall and Trivelpiece (1973). The result is

$$mn\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right] = qn(\mathbf{E} + \mathbf{u}\times\mathbf{B}) - \nabla\cdot\mathbf{\Pi} + \mathbf{f}\Big|_c \qquad (2.3.9)$$

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Momentum Conservation

The left-hand side is the species mass density times the convective derivative of the mean velocity, representing the mass density times the acceleration. The convective derivative has two terms: the first term $\partial \mathbf{u}/\partial t$ represents an acceleration due to an explicitly time-varying $\mathbf{u}$; the second "inertial" term $(\mathbf{u} \cdot \nabla)\mathbf{u}$ represents an acceleration even for a steady fluid flow ($\partial/\partial t \equiv 0$) having a spatially varying $\mathbf{u}$. For example, if $\mathbf{u} = \hat{x}\, u_x(x)$ increases along x, then the fluid is accelerating along x (Problem 2.4). This second term is nonlinear in $\mathbf{u}$ and can often be neglected in discharge analysis.

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The mass times acceleration is acted upon, on the right-hand side, by the body forces, with the first term being the electric and magnetic force densities. The second term is the force density due to the divergence of the pressure tensor, which arises due to the integration over velocities

$$\Pi_{ij} = mn\langle (v_i - u)(v_j - u)\rangle_{\mathbf{v}} \tag{2.3.10}$$

where the subscripts i, j give the component directions and $\langle\cdot\rangle_{\mathbf{v}}$ denotes the velocity average of the bracketed quantity over f.* For weakly ionized plasmas it is almost never used in this form, but rather an isotropic version is employed:

$$\boldsymbol{\Pi} = \begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix} \tag{2.3.11}$$

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FIELD EQUATIONS, CURRENT, AND VOLTAGE

such that

Momentum Conservation

$$\nabla \cdot \boldsymbol{\Pi} = \nabla p \tag{2.3.12}$$

a pressure gradient, with

$$p = \frac{1}{3} mn \langle (v-u)^2 \rangle_{\mathbf{v}} \tag{2.3.13}$$

being the scalar pressure. Physically, the pressure gradient force density arises as illustrated in Figure 2.4, which shows a small volume acted upon by a pressure that is an increasing function of x. The net force on this volume is $p(x)\,dA - p(x + dx)\,dA$ and the volume is $dA\,dx$. Hence the force per unit volume is $-\partial p/\partial x$.

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THE CONSERVATION EQUATIONS

Momentum Conservation

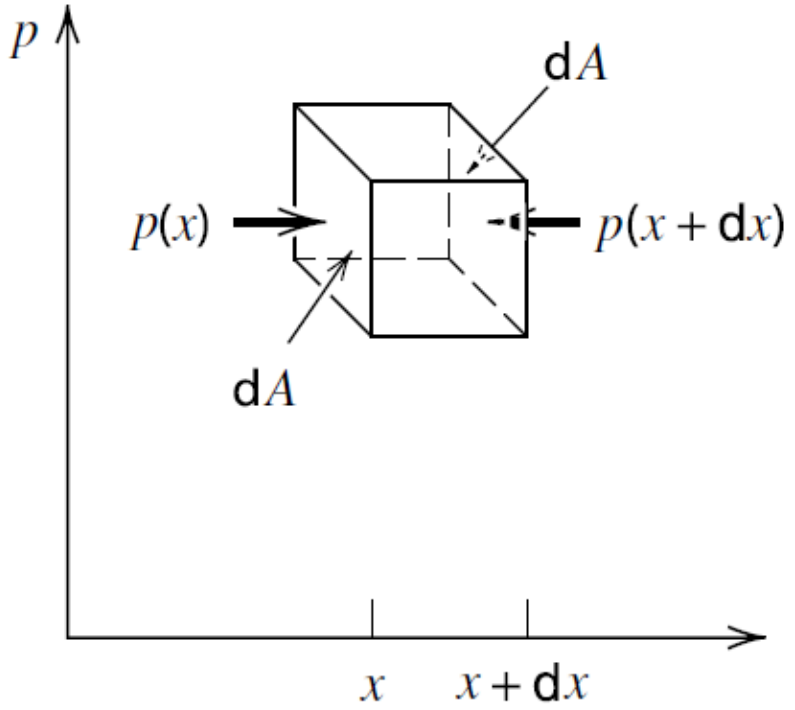


FIGURE 2.4. The force density due to the pressure gradient.

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Basic Plasma Equations and Equilibrium

THE CONSERVATION EQUATIONS

Momentum Conservation

The third term on the right in (2.3.9) represents the time rate of momentum transfer per unit volume due to collisions with other species. For electrons or positive ions the most important transfer is often due to collisions with neutrals. The transfer is usually approximated by a Krook collision operator

$$\mathbf{f}|_{\mathrm{c}} = -\sum_{\beta} mn\nu_{\mathrm{m}\beta}(\mathbf{u}-\mathbf{u}_{\beta}). - m(\mathbf{u}-\mathbf{u}_{\mathrm{G}})G + m(\mathbf{u}-\mathbf{u}_{\mathrm{L}})L \qquad (2.3.14)$$

where the summation is over all other species, $\mathbf{u}_{\beta}$ is the mean velocity of species β, $\nu_{\mathrm{m}\beta}$ is the momentum transfer frequency for collisions with species β, and $\mathbf{u}_{\mathrm{G}}$ and $\mathbf{u}_{\mathrm{L}}$ are the mean velocities of newly created and lost particles. Generally $|\mathbf{u}_{\mathrm{G}}| \ll |\mathbf{u}|$ for

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pair creation by ionization, and $\mathbf{u}_L \approx \mathbf{u}$ for recombination or charge transfer loss processes. We discuss the Krook form of the collision operator further in Chapter 18. The last two terms in (2.3.14) are generally small and give the momentum transfer due to the creation or destruction of particles. For example, if ions are created at rest, then they exert a drag force on the moving ion fluid because they act to lower the average fluid velocity.

A common form of the average force (momentum conservation) equation is obtained from (2.3.9) neglecting the magnetic forces and taking $\mathbf{u}_\beta = 0$ in the Krook collision term for collisions with one neutral species. The result is

$$mn\left[\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right] = qn\mathbf{E} - \nabla p - mn\nu_m\mathbf{u} \qquad (2.3.15)$$

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Momentum Conservation

$$mn\left[\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right] = qn\mathbf{E} - \nabla p - mn\nu_m\mathbf{u} \qquad (2.3.15)$$

where only the acceleration $(\partial\mathbf{u}/\partial t)$, inertial $(\mathbf{u}\cdot\nabla\mathbf{u})$, electric field, pressure gradient, and collision terms appear. For slow time variation, the acceleration term can be neglected. For high pressures, the inertial term is small compared to the collision term and can also be dropped.

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THE CONSERVATION EQUATIONS

Momentum Conservation

Equations (2.3.7) and (2.3.9) together still do not form a closed set, since the pressure tensor $\mathbf{\Pi}$ (or scalar pressure p) is not determined. The usual procedure to close the equations is to use a thermodynamic *equation of state* to relate p to n. The *isothermal* relation for an equilibrium Maxwellian distribution is

$$p = nkT \tag{2.3.16}$$

so that

$$\nabla p = kT\nabla n \tag{2.3.17}$$

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where T is the temperature in kelvin and k is Boltzmann's constant ($k = 1.381 \times 10^{-23}$ J/K). This holds for slow time variations, where temperatures are allowed to equilibrate. In this case, the fluid can exchange energy with its surroundings, and we also require an energy conservation equation (see below) to determine p and T. For a room temperature (297 K) neutral gas having density n_g and pressure p, (2.3.16) yields

$$n_g\ (\text{cm}^{-3}) \approx 3.250 \times 10^{16}\, p\,(\text{Torr}) \qquad (2.3.18)$$

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Alternatively, the *adiabatic* equation of state is

$$p = Cn^{\gamma} \tag{2.3.19}$$

such that

$$\frac{\nabla p}{p} = \gamma \frac{\nabla n}{n} \tag{2.3.20}$$

where γ is the ratio of specific heat at constant pressure to that at constant volume. The specific heats are defined in Section 7.2; $\gamma = 5/3$ for a perfect gas; for one-dimensional adiabatic motion, $\gamma = 3$. The adiabatic relation holds for fast time variations, such as in waves, when the fluid does not exchange energy with its surroundings; hence an energy conservation equation is not required. For almost all applications to discharge analysis, we use the isothermal equation of state.

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Energy Conservation

$$mn\left[\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right] = qn\mathbf{E} - \nabla p - mn\nu_m\mathbf{u} \qquad (2.3.15)$$

where only the acceleration $(\partial\mathbf{u}/\partial t)$, inertial $(\mathbf{u}\cdot\nabla\mathbf{u})$, electric field, pressure gradient, and collision terms appear. For slow time variation, the acceleration term can be neglected. For high pressures, the inertial term is small compared to the collision term and can also be dropped.

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Energy Conservation

The energy conservation equation is obtained by multiplying the Boltzmann equation by $\frac{1}{2}mv^2$ and integrating over velocity. The integration and some other manipulation yield

$$\frac{\partial}{\partial t}\left(\frac{3}{2}p\right) + \nabla \cdot \frac{3}{2}(p\mathbf{u}) + p\nabla \cdot \mathbf{u} + \nabla \cdot \mathbf{q} = \frac{\partial}{\partial t}\left(\frac{3}{2}p\right)\bigg|_c \qquad (2.3.21)$$

Here $\frac{3}{2}p$ is the thermal energy density ($\mathrm{J/m^3}$), $\frac{3}{2}p\mathbf{u}$ is the macroscopic thermal energy flux ($\mathrm{W/m^2}$), representing the flow of the thermal energy density at the fluid velocity $\mathbf{u}$, $p\nabla \cdot \mathbf{u}$ ($\mathrm{W/m^3}$) gives the heating or cooling of the fluid due to compression or expansion of its volume (Problem 2.5), $\mathbf{q}$ is the heat flow vector ($\mathrm{W/m^2}$), which gives the microscopic thermal energy flux, and the collisional term includes all collisional processes that change the thermal energy density. These include ionization,

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excitation, elastic scattering, and frictional (ohmic) heating. The equation is usually closed by setting $\mathbf{q} = 0$ or by letting $\mathbf{q} = -\kappa_T \nabla T$, where κ_T is the thermal conductivity. For most steady-state discharges the macroscopic thermal energy flux is balanced against the collisional processes, giving the simpler equation

$$\nabla \cdot \left(\frac{3}{2} p \mathbf{u}\right) = \frac{\partial}{\partial t}\left(\frac{3}{2} p\right)\bigg|_c \tag{2.3.22}$$

Equation (2.3.22), together with the continuity equation (2.3.8), will often prove sufficient for our analysis.

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THE CONSERVATION EQUATIONS

Summary

Summarizing our results for the macroscopic equations describing the electron and ion fluids, we have in their most usually used forms the continuity equation

$$\nabla \cdot (n\mathbf{u}) = \nu_{iz} n_e \tag{2.3.8}$$

the force equation,

$$mn\left[\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right] = qn\mathbf{E} - \nabla p - mn\nu_m \mathbf{u} \tag{2.3.15}$$

the isothermal equation of state

$$p = nkT \tag{2.3.16}$$

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THE CONSERVATION EQUATIONS

Summary

and the energy-conservation equation

$$\nabla \cdot \left(\frac{3}{2} p\mathbf{u}\right) = \frac{\partial}{\partial t}\left(\frac{3}{2} p\right)\bigg|_{\mathrm{c}} \tag{2.3.22}$$

These equations hold for each charged species, with the total charges and currents summed in Maxwell's equations. For example, with electrons and one positive ion species with charge Ze, we have

$$\rho = e(Zn_{\mathrm{i}} - n_{\mathrm{e}}) \tag{2.3.23}$$

$$\mathbf{J} = e(Zn_{\mathrm{i}}\mathbf{u}_{\mathrm{i}} - n_{\mathrm{e}}\mathbf{u}_{\mathrm{e}}) \tag{2.3.24}$$

These equations are still very difficult to solve without simplifications. They consist of 18 unknown quantities $n_{\mathrm{i}}, n_{\mathrm{e}}, p_{\mathrm{i}}, p_{\mathrm{e}}, T_{\mathrm{i}}, T_{\mathrm{e}}, \mathbf{u}_{\mathrm{i}}, \mathbf{u}_{\mathrm{e}}, \mathbf{E}$, and $\mathbf{B}$, with the vectors each counting for three. Various simplifications used to make the solutions to the equations tractable will be employed as the individual problems allow.

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Electrons are generally in near-thermal equilibrium at temperature T_e in discharges, whereas positive ions are *almost never* in thermal equilibrium. Neutral gas molecules may or may not be in thermal equilibrium, depending on the generation and loss processes. For a single species in thermal equilibrium with itself (e.g., electrons), in the absence of time variation, spatial gradients, and accelerations, the Boltzmann equation (2.3.3) reduces to

$$\left.\frac{\partial f}{\partial t}\right|_c = 0 \qquad (2.4.1)$$

where the subscript c here represents the collisions of a particle species with itself.

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We show in Appendix B that the solution of (2.4.1) has a Gaussian speed distribution of the form

$$f(v) = C\,\mathrm{e}^{-\xi^2 m v^2} \tag{2.4.2}$$

The two constants C and ξ can be obtained by using the thermodynamic relation

$$w = \frac{1}{2} mn\langle v^2 \rangle_{\mathbf{v}} = \frac{3}{2} nkT \tag{2.4.3}$$

that is, that the average energy of a particle is $\frac{1}{2}kT$ per translational degree of freedom, and by using a suitable normalization of the distribution. Normalizing

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$f(v)$ to n, we obtain

$$C\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^{\infty} \exp\left(-\xi^2 m v^2\right) v^2\, dv = n \tag{2.4.4}$$

and using (2.4.3), we obtain

$$\frac{1}{2} mC\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^{\infty} \exp\left(-\xi^2 m v^2\right) v^4 dv = \frac{3}{2} nkT \tag{2.4.5}$$

where we have written the integrals over velocity space in spherical coordinates. The angle integrals yield the factor 4π. The v integrals are evaluated using the relation*

$$\int_0^{\infty} e^{-u^2} u^{2i}\, du = \frac{(2i-1)!!}{2^{i+1}} \sqrt{\pi}, \quad \text{where } i \text{ is an integer} \geq 1. \tag{2.4.6}$$

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Solving for C and ξ we have

$$f(v) = n\left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) \tag{2.4.7}$$

which is the *Maxwellian distribution.*

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Similarly, other averages can be performed. The average speed $\bar{v}$ is given by

$$\bar{v} = (m/2\pi kT)^{3/2} \int_0^\infty v \left[\exp\left(-\frac{v^2}{2v_{\text{th}}^2} \right) \right] 4\pi v^2 \, dv \qquad (2.4.8)$$

where $v_{\text{th}} = (kT/m)^{1/2}$ is the thermal velocity. We obtain

$$\bar{v} = \left(\frac{8kT}{\pi m} \right)^{1/2} \qquad (2.4.9)$$

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The directed flux Γ_z in (say) the $+z$ direction is given by $n\langle v_z\rangle_{\mathbf{v}}$, where the average is taken over $v_z > 0$ only. Writing $v_z = v\cos\theta$ we have in spherical coordinates

$$\Gamma_z = n\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^{2\pi} d\phi \int_0^{\pi/2} \sin\theta\, d\theta \int_0^{\infty} v\cos\theta \exp\left(-\frac{v^2}{2v_{\text{th}}^2}\right) v^2\, dv$$

Evaluating the integrals, we find

$$\Gamma_z = \frac{1}{4}n\bar{v} \qquad (2.4.10)$$

Γ_z is the number of particles per square meter per second crossing the $z = 0$ surface in the positive direction. Similarly, the average energy flux $S_z = n\langle \frac{1}{2}mv^2 v_z\rangle_{\mathbf{v}}$ in the $+z$ direction can be found: $S_z = 2kT\Gamma_z$. We see that the average kinetic energy W per

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particle crossing $z = 0$ in the positive direction is

$$W = 2kT \tag{2.4.11}$$

It is sometimes convenient to define the distribution in terms of other variables. For example, we can define a distribution of energies $W = \frac{1}{2}mv^2$ by

$$4\pi g(W)\,dW = 4\pi f(v)v^2\,dv$$

Evaluating dv/dW, we see that g and f are related by

$$g(W) = \frac{v(W)f[v(W)]}{m} \tag{2.4.12}$$

4Pi

where $v(W) = (2W/m)^{1/2}$.

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Basic Plasma Equations and Equilibrium

Boltzmann's Relation

A very important relation can be obtained for the density of electrons in thermal equilibrium at varying positions in a plasma under the action of a spatially varying potential. In the absence of electron drifts ($\mathbf{u}_e \equiv 0$), the inertial, magnetic, and frictional forces are zero, and the electron force balance is, from (2.3.15) with $\partial/\partial t \equiv 0$,

$$en_e\mathbf{E} + \nabla p_e = 0 \tag{2.4.13}$$

Setting $\mathbf{E} = -\nabla\Phi$ and assuming $p_e = n_e kT_e$, (2.4.13) becomes

$$-en_e\nabla\Phi + kT_e\nabla n_e = 0$$

or, rearranging,

$$\nabla(e\Phi - kT_e \ln n_e) = 0 \tag{2.4.14}$$

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Basic Plasma Equations and Equilibrium

Boltzmann's Relation

Integrating, we have

$$e\Phi - kT_e \ln n_e = \text{const}$$

or

$$n_e(\mathbf{r}) = n_0 e^{e\Phi(\mathbf{r})/kT_e} \qquad (2.4.15)$$

which is *Boltzmann's relation* for electrons. We see that electrons are "attracted" to regions of positive potential. We shall generally write Boltzmann's relation in more convenient units

$$n_e = n_0 e^{\Phi/T_e} \qquad (2.4.16)$$

where T_e is now expressed in volts, as is Φ.

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Basic Plasma Equations and Equilibrium

Boltzmann's Relation

For positive ions in thermal equilibrium at temperature T_i, a similar analysis shows that

$$n_i = n_0 e^{-\Phi/T_i} \qquad (2.4.17)$$

Hence positive ions in thermal equilibrium are "repelled" from regions of positive potential. However, positive ions are almost never in thermal equilibrium in low-pressure discharges because the ion drift velocity $\mathbf{u}_i$ is large, leading to inertial or frictional forces in (2.3.15) that are comparable to the electric field or pressure gradient forces.

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Basic Plasma Equations and Equilibrium

Debye Length

The characteristic length scale in a plasma is the electron Debye length λ_{De}. As we will show, the Debye length is the distance scale over which significant charge densities can spontaneously exist. For example, low-voltage (undriven) sheaths are typically a few Debye lengths wide. To determine the Debye length, let us introduce a sheet of negative charge having surface charge density $\rho_s < 0$ C/m^2 into an infinitely extended plasma having equilibrium densities $n_e = n_i = n_0$. For simplicity we assume immobile ions, such that $n_i = n_0$ after the sheet is introduced. However, the negative sheet "repels" nearby electrons, leading to a reduced electron density near the sheet. The situation after introduction of the sheet is shown in Figure 2.5. To determine the potential and density variation, we use Poisson's equation, which in one dimension can be written as

$$\frac{d^2\Phi}{dx^2} = -\frac{e}{\epsilon_0}(n_i - n_e) \qquad (2.4.18)$$

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Debye Length

Setting $n_e = n_0 \exp(\Phi/T_e)$, from the Boltzmann relation (2.4.16), and taking $n_i = n_0$, Poisson's equation becomes

$$\frac{d^2\Phi}{dx^2} = \frac{en_0}{\epsilon_0}\left(e^{\Phi/T_e} - 1\right) \tag{2.4.19}$$

Expanding $\exp(\Phi/T_e)$ in a Taylor series for $\Phi \ll T_e$, (2.4.19) becomes, to lowest order in Φ/T_e,

$$\frac{d^2\Phi}{dx^2} = \frac{en_0}{\epsilon_0}\frac{\Phi}{T_e} \tag{2.4.20}$$

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The symmetric solution of (2.4.20) that vanishes at $x = \pm\infty$ is

$$\Phi = \Phi_0 \, e^{-|x|/\lambda_{De}} \tag{2.4.21}$$

where

$$\lambda_{De} = \left(\frac{\epsilon_0 T_e}{e n_0}\right)^{1/2} \tag{2.4.22}$$

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Debye Length

In practical units, we find

$$\lambda_{De}\ (\text{cm}) \approx 743\sqrt{T_e/n_e} \tag{2.4.23}$$

with T_e in volts and n_e in cm^{-3}. We find for $T_e = 4\ \text{V}$ and $n_e = 10^{10}\ \text{cm}^{-3}$ that $\lambda_{De} = 0.14\ \text{mm}$. It is on space scales larger than a Debye length that the plasma will tend to remain neutral.

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Debye Length

The Debye length is useful in many contexts. In the next chapter we shall see that it serves as a characteristic scale length to shield the Coulomb potentials of individual charged particles when they collide. Although we have calculated the above effect for electron shielding, it is also possible on slower time scales for the ions to contribute. We leave the calculation for a problem. Ion shielding plays a key role in dusty plasmas, which we treat in Chapter 17.

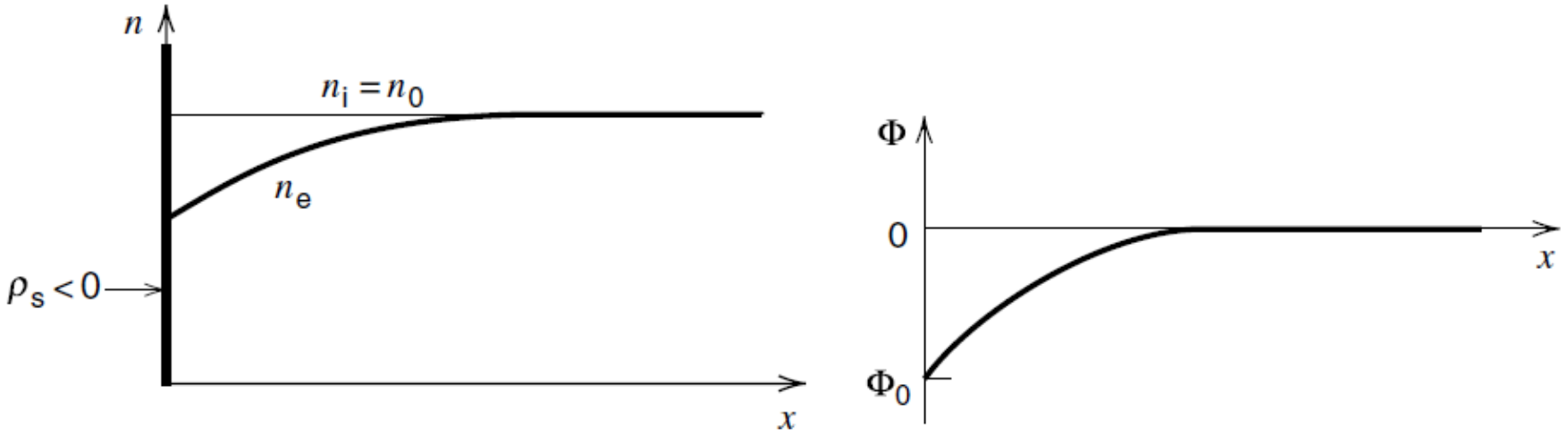


FIGURE 2.5. Calculation of the electron Debye length λ_{De}. A negatively charged sheet is introduced into a plasma containing electrons in thermal equilibrium.

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Quasi-neutrality

The potential variation across a plasma of length $l \gg \lambda_{De}$ can be estimated from Poisson's equation (2.2.11):

$$\nabla^2\Phi \sim \frac{\Phi}{l^2} \sim \left|\frac{e}{\epsilon_0}(Zn_i - n_e)\right| \tag{2.4.24}$$

We generally expect that

$$\Phi \lesssim T_e = \frac{e}{\epsilon_0} n_e \lambda_{De}{}^2 \tag{2.4.25}$$

where the equality on the right follows from the definition of λ_{De}. Combining (2.4.24) and (2.4.25) we have

$$\frac{|Zn_i - n_e|}{n_e} \lesssim \frac{\lambda_{De}^2}{l^2} \tag{2.4.26}$$

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Basic Plasma Equations and Equilibrium

Quasi-neutrality

For $\lambda_{De}^2/l^2 \ll 1$, (2.4.26) implies that

$$|Zn_i - n_e| \ll n_e \tag{2.4.27}$$

such that we can set

$$Zn_i = n_e \tag{2.4.28}$$

except when used in Poisson's equation. Relation (2.4.27) is the basic statement of *quasi-neutrality* of a plasma, and is often called the *plasma approximation*. We shall see in Chapter 6 that the plasma approximation is violated within a plasma sheath, in proximity to a material wall, either because the sheath thickness $s \approx \lambda_{De}$, or because $\Phi \gg T_e$.

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